



Calculation Policy

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Date:	May 2018
Ratified by Governors:	Delegated to Head Teacher

Progression Towards a Written Method for Addition

In developing a written method for addition, it is important that children understand the concept of addition, in that it is:

- Combining two or more groups to give a total or sum
- Increasing an amount

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of subtraction
- commutative i.e. $5 + 3 = 3 + 5$
- associative i.e. $5 + 3 + 7 = 5 + (3 + 7)$

The fact that it is commutative and associative means that calculations can be rearranged, e.g.

$4 + 13 = 17$ is the same as $13 + 4 = 17$.

EYFS

Early Learning Goal:

Using quantities and objects, children add two single-digit numbers and count on to find the answer.

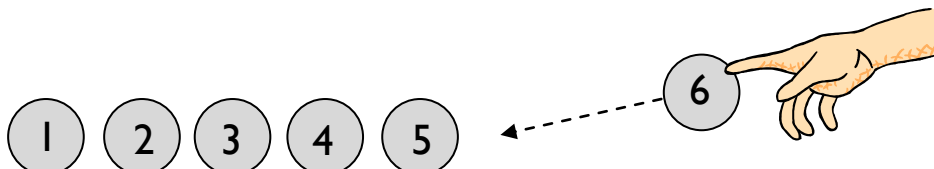
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of practical equipment, including small world play, role play, counters, cubes etc.

Counting all method

Children will begin to develop their ability to add by using practical equipment to count out the correct amount for each number in the calculation and then combine them to find the total. For example, when calculating $4 + 2$, they are encouraged to count out four counters and count out two counters.



To find how many altogether, touch and drag them into a line one at a time whilst counting.



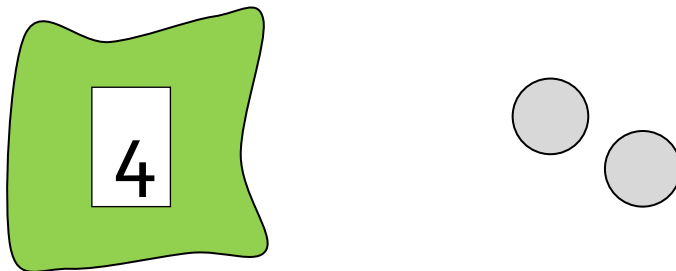
By touch counting and dragging in this way, it allows children to keep track of what they have already counted to ensure they don't count the same item twice.

Counting on method

To support children in moving from a counting all strategy to one involving counting on, children should still have two groups of objects but one should be covered so that it cannot be counted. For example, when calculating $4 + 2$, count out the two groups of counters as before.



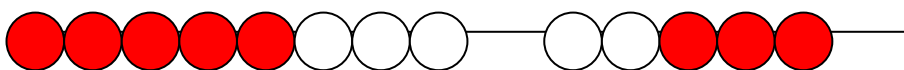
then cover up the larger group with a cloth.



For most children, it is beneficial to place the digit card on top of the cloth to remind the children of the number of counters underneath. They can then start their count at 4, and touch count 5 and 6 in the same way as before, rather than having to count all of the counters separately as before.

Those who are ready may record their own calculations.

Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.



e.g. $8 + 5 =$

Y1

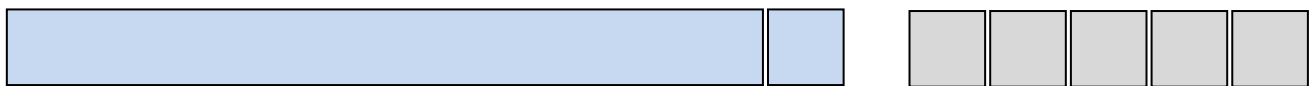
End of Year Objective:

Add one-digit and two-digit numbers to 20, including zero (using concrete objects and pictorial representations).

Children will continue to use practical equipment, combining groups of objects to find the total by counting all or counting on. Using their developing understanding of place value,

they will move on to be able to use Base 10 equipment to make teens numbers using separate tens and ones.

For example, when adding 11 and 5, they can make the 11 using a ten rod and a one.

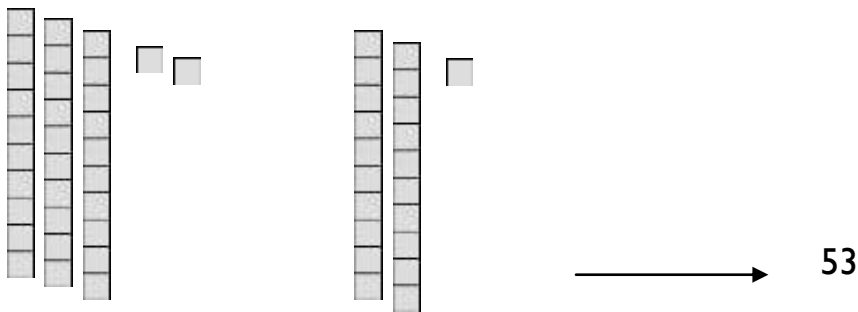


The ones can then be combined to aid with seeing the final total, e.g.



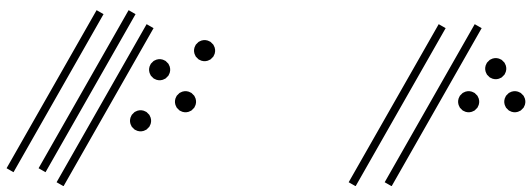
so $11 + 5 = 16$. If possible, they should use two different colours of base 10 equipment so that the initial amounts can still be seen.

Children will continue to use the Base 10 equipment to support their calculations for two digit numbers. For example, to calculate $32 + 21$, they can make the individual amounts, counting the tens first and then count on the ones.



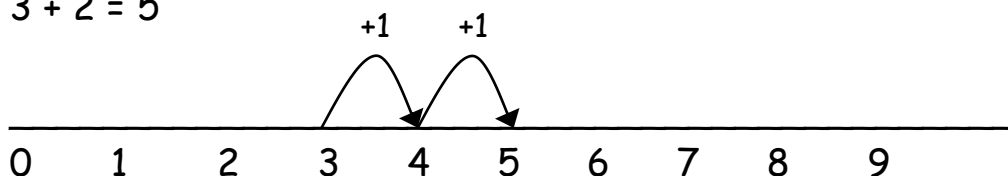
Children can also record the calculations using their own drawings of the Base 10 equipment (as slanted lines for the 10 rods and dots for the one blocks).

e.g. $34 + 23 =$



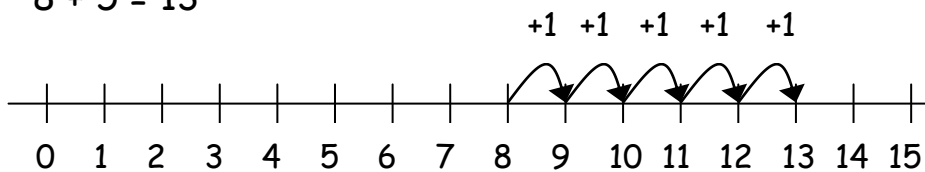
They will use number-lines and practical resources to support calculation and teachers *demonstrate* the use of the number-line.

$$3 + 2 = 5$$



Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.

$$8 + 5 = 13$$



Y2

End of Year Objective:

Add numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers; three one-digit numbers.

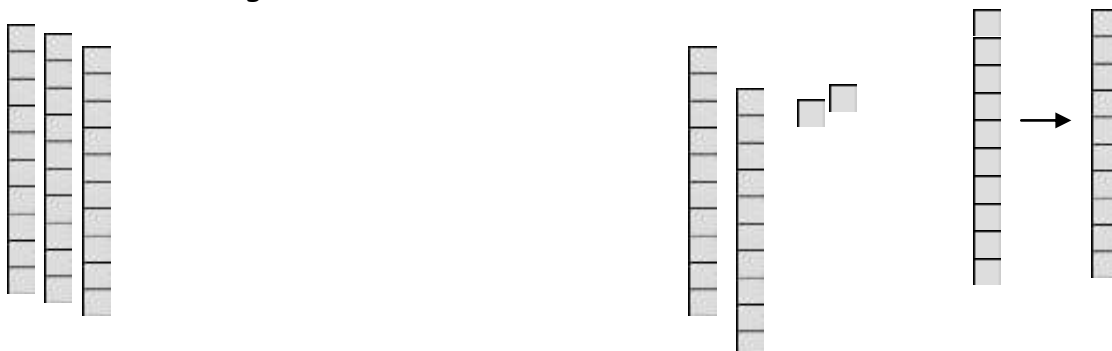
Children will continue to develop their understanding of addition using the Base 10 equipment. They will increase their understanding of what happens when the ones total more than 10, as shown.

When the ones total more than 10, children should be encouraged to exchange 10 ones for 1 ten. This is the start of children understanding 'carrying' in vertical addition. For example, when calculating

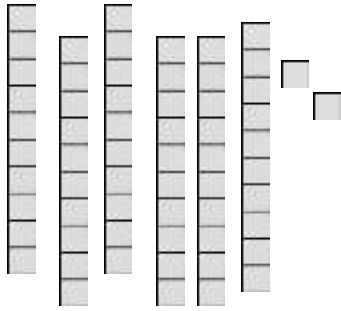
$35 + 27$, they can represent the amounts using Base 10 as shown:



Then, identifying the fact that there are enough ones to exchange for a ten, they can carry out this exchange:

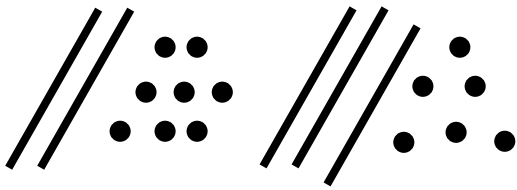


To leave:

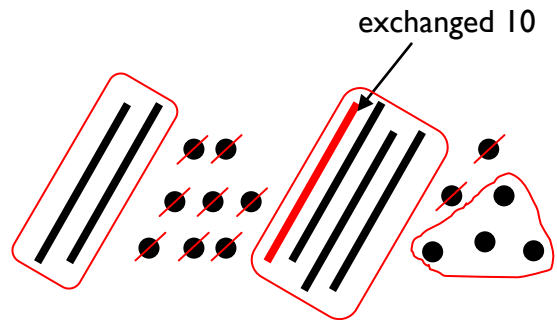


With exchange:

e.g. $28 + 36 =$



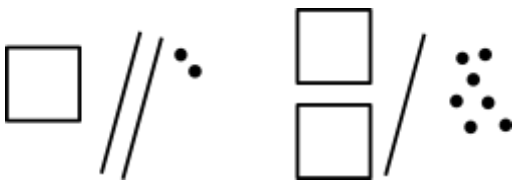
will become



so $28 + 36 = 64$

It is important that children circle the remaining tens and ones after exchange to identify the amount remaining.

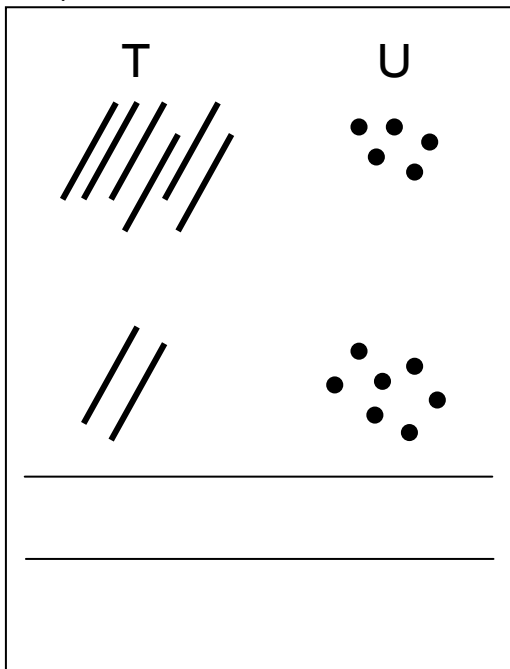
This method can also be used with adding three digit numbers, e.g. $122 + 217$ using a square as the representation of 100.



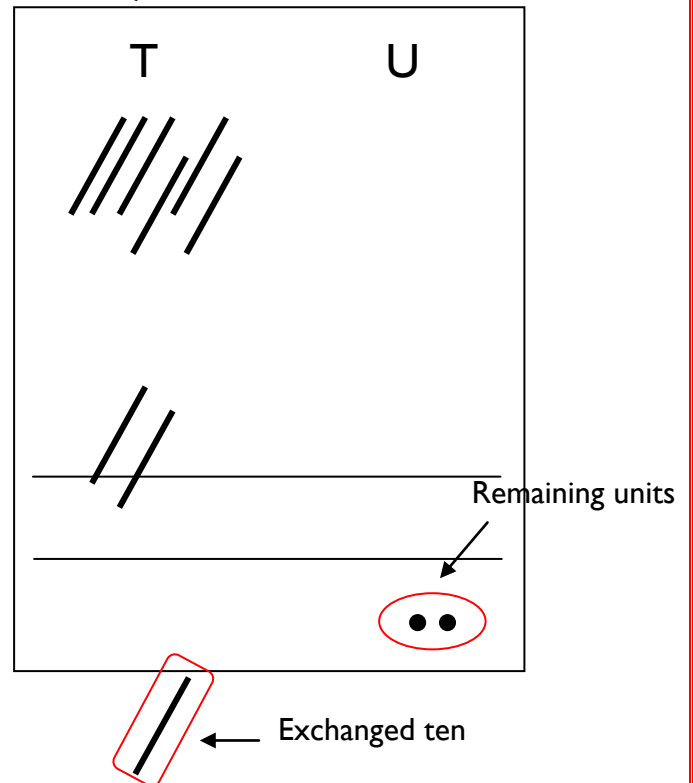
They can use a place value grid to begin to set the calculation out vertically and to support their knowledge of exchange between columns (as in Step 1 in the diagram below).

e.g. $65 + 27$

Step 1



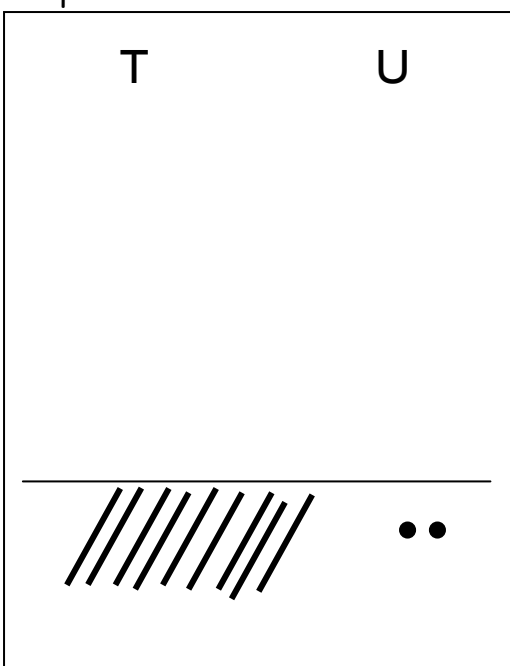
Step 2



Children would exchange ten ones for a ten, placing the exchanged ten below the equals sign. Any remaining ones that cannot be exchanged for a ten move into the equals sign as they are the ones part of the answer (as in the diagram in Step 2 above).

The tens move into the equals sign as they are the tens part of the answer (as in the diagram in Step 3 below).

Step 3



Written method

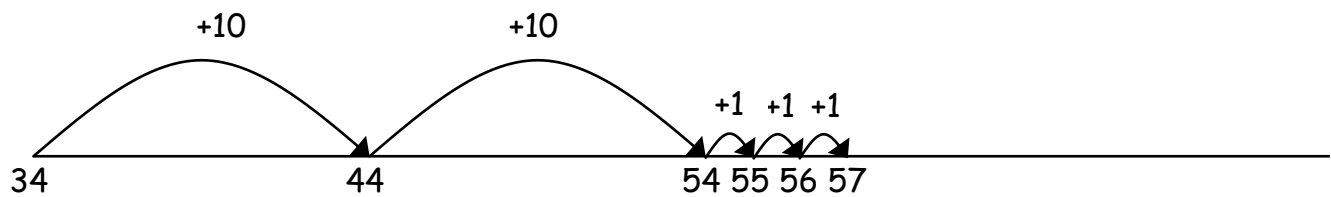
Step 1	Step 2	Step 3																																						
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Children should utilise this practical method to link their understanding of exchange to how the column method is set out. Teachers should model the written method alongside this practical method initially. This should progress to children utilising the written and practical methods alongside each other and finally, and when they are ready, to children utilising just the written method.

Children will begin to use 'empty number lines' themselves, starting with the larger number and counting on.

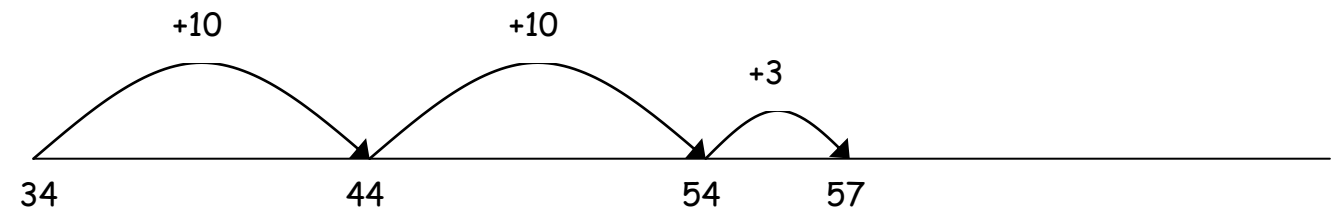
- ✓ First counting on in tens and ones.

$$34 + 23 = 57$$



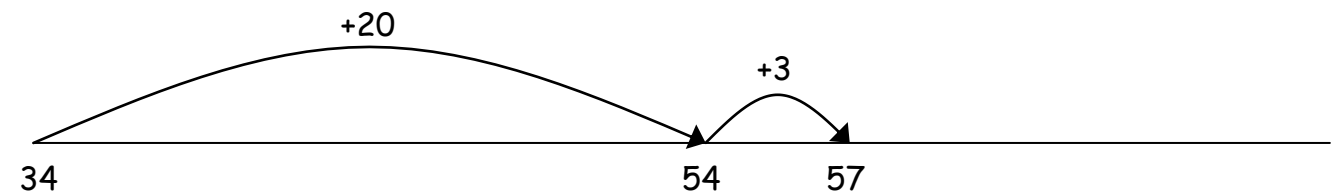
- ✓ Then helping children to become more efficient by adding the ones in one jump (by using the known fact $4 + 3 = 7$).

$$34 + 23 = 57$$



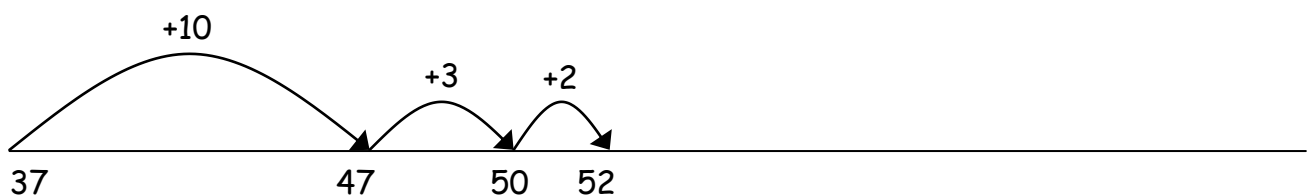
- ✓ Followed by adding the tens in one jump and the ones in one jump.

$$34 + 23 = 57$$



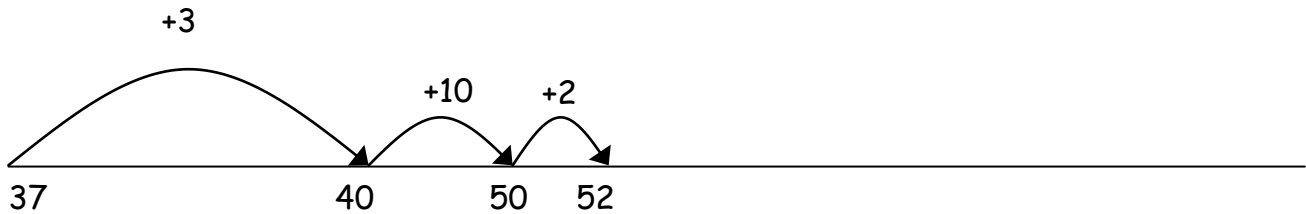
- ✓ Bridging through ten can help children become more efficient.

$$37 + 15 = 52$$



- ✓ Finding the next friendly number can help children find the difference or solve empty number box calculations.

$$37 + ? = 52$$



Y3

End of Year Objective:
Add numbers with up to three digits, using formal written method of columnar addition.

Children will continue to use the written column method, supported with Base 10 if required.

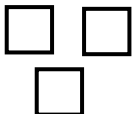

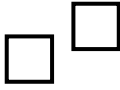

Children will use Base 10 or jottings to exchange ten tens for a hundred. As in year 2, they should be comfortable with exchanging practically before they move onto using the formal column method for addition.

Step 1

H	T	U

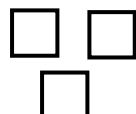
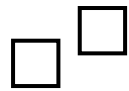

H	T	U
3	6	5
+	2	4

Step 2

H	T	U
		
		
		••
		/

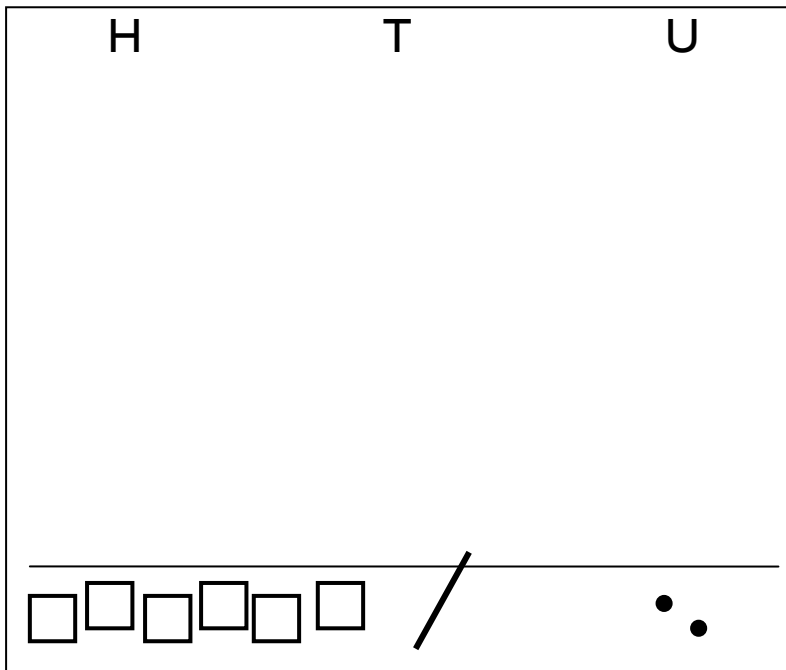
H	T	U
3	6	5
+	2	4
		7
		2

Step 3

H	T	U
		
		
		••
		/

H	T	U
3	6	5
+	2	4
		7
		2

Step 4



	H	T	U
	3	6	5
+	2	4	7
	6	1	2
	1	1	

From this, children will begin to carry below the line.

$$\begin{array}{r} 625 \\ + 48 \\ \hline 673 \\ 1 \end{array}$$

$$\begin{array}{r} 783 \\ + 42 \\ \hline 825 \\ 1 \end{array}$$

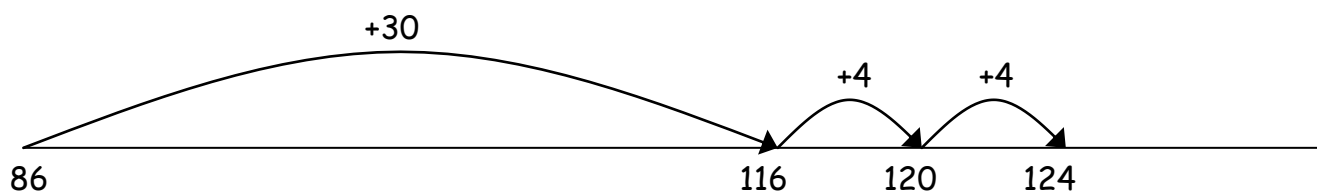
$$\begin{array}{r} 367 \\ + 85 \\ \hline 452 \\ 11 \end{array}$$

By mid-year 3, children should be able to use the column method sufficiently up to three digits with exchange in both the tens and ones column (as per the last example above.)

Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate.

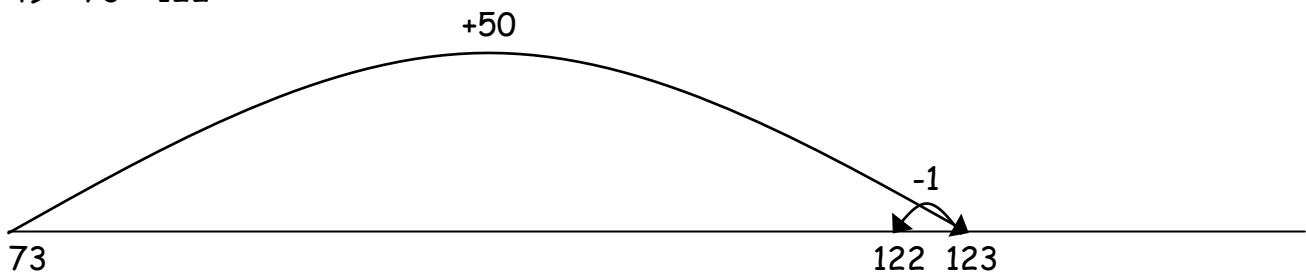
- ✓ Count on from the largest number irrespective of the order of the calculation.

$$38 + 86 = 124$$



✓ Compensation

$$49 + 73 = 122$$



Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

Y4

End of Year Objective:

Add numbers with up to 4 digits *and decimals with one decimal place* using the formal written method of columnar addition where appropriate.

Children will move to year 4 using column method. They should also be able to draw jottings in columns *if necessary* to develop and consolidate their understanding.

By the end of year 4, children should be using the written method to add:

- several numbers with **different numbers** of digits, understanding the place value;
- *decimals with one decimal place, knowing that the decimal points line up under one another.*
- *begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds;*
- *know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + 78p.*

Y5

End of Year Objective:

Add whole numbers with more than 4 digits *and decimals with two decimal places*, including formal written methods (columnar addition).

Children should continue to use the carrying method to solve calculations such as:

$$\begin{array}{r} 3364 \\ + 247 \\ \hline 3611 \\ \hline | \quad | \end{array}$$

$$\begin{array}{r} 3121 \\ + 148 \\ \hline 3306 \\ \hline | \quad | \end{array}$$

$$\begin{array}{r} 3.56 \\ + 2.47 \\ \hline 6.03 \\ \hline | \end{array}$$

They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with each number having the same number of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another

Y6

End of Year Objective:

Add whole numbers and decimals using formal written methods (columnar addition).

Children should extend the carrying method and use it to add whole numbers and decimals with any number of digits.

$$\begin{array}{r} \\ \\ \\ \\ \\ + \\ \hline 1 \\ \hline 1 \end{array}$$

$$\begin{array}{r} \\ \\ \\ \\ \\ + \\ \hline 4 \\ \hline 4 \end{array}$$

When adding decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.

They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with mixed numbers of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another.

Progression Towards a Written Method for Subtraction

In developing a written method for subtraction, it is important that children understand the concept of subtraction, in that it is:

- Removal of an amount from a larger group (take away)
- Comparison of two amounts (difference)

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of addition
- not commutative i.e. $5 - 3$ is not the same as $3 - 5$
- not associative i.e. $10 - 3 - 2$ is not the same as $10 - (3 - 2)$

EYFS

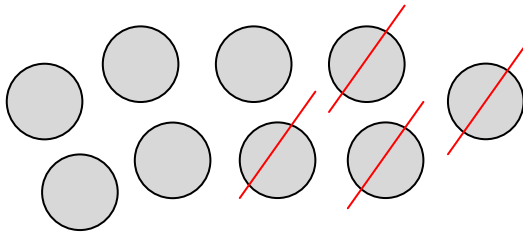
Early Learning Goal:

Using quantities and objects, children subtract two single-digit numbers and count on or back to find the answer.

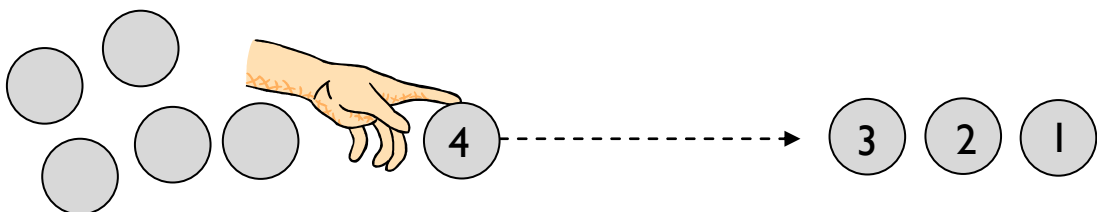
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of practical equipment, including small world play, role play, counters, cubes etc.

Taking away

Children will begin to develop their ability to subtract by using practical equipment to count out the first number and then remove or take away the second number to find the solution by counting how many are left e.g. $9 - 4$.



For illustration purposes, the amount being taken away are show crossed out. Children would be encouraged to physically remove these using touch counting.

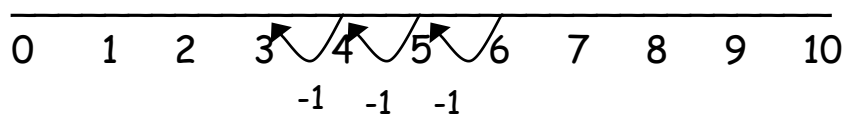


By touch counting and dragging in this way, it allows children to keep track of how many they are removing so they don't have to keep recounting. They will then touch count the amount that are left to find the answer.

Those who are ready may record their own calculations.

They use numberlines, number tracks and practical resources to support calculation. Teachers *demonstrate* the use of the numberline.

$$6 - 3 = 3$$

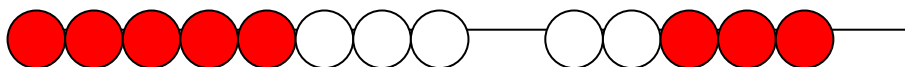


The numberline should also be used to show that $6 - 3$ means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart.



Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.

$$13 - 5 = 8$$



Y1

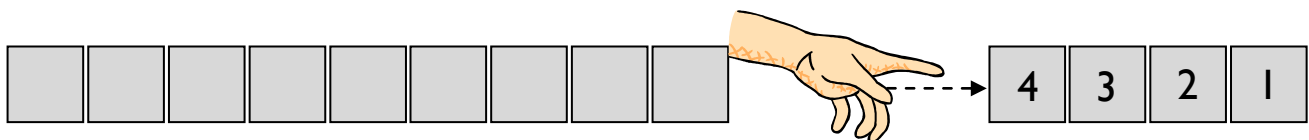
End of Year Objective:

Subtract one-digit and two-digit numbers to 20, including zero (using concrete objects and pictorial representations).

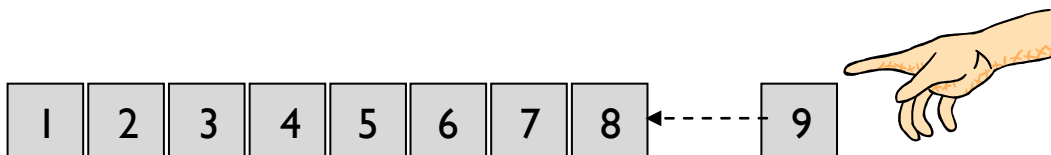
Children will continue to use practical equipment and taking away strategies. To avoid the need to exchange for subtraction at this stage, it is advisable to continue to use equipment such as counters, cubes and the ones from the Base 10 equipment, but not the tens, e.g. $13 - 4$



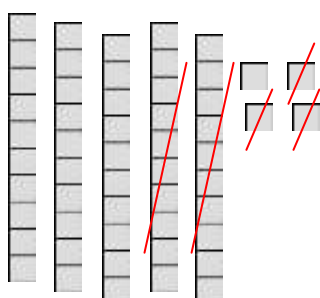
Touch count and remove the number to be taken away, in this case 4.



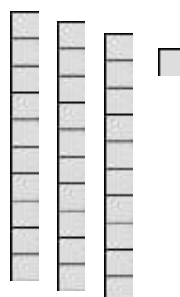
Touch count to find the number that remains.



Children will begin to use the Base 10 equipment to support their calculations, still using a take away, or removal, method. They need to understand that the number being subtracted does not appear as an amount on its own, but rather as part of the larger amount. For example, to calculate $54 - 23$, children would count out 54 using the Base 10 equipment (5 tens and 4 ones). They need to consider whether there are enough ones to remove 3, in this case there are, so they would remove 3 ones and then two tens, counting up the answer of 3 tens and 1 one to give 31.



which leaves



so $54 - 23 = 31$

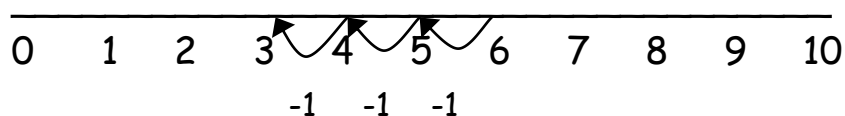
Children can also record the calculations using their own drawings of the Base 10 equipment (as slanted lines for the 10 rods and dots for the one blocks), e.g. to calculate $39 - 17$ children would draw 39 as 3 tens (lines) and 4 ones (dots) and would cross out 7 ones and then one ten, counting up the answer of 2 tens and 2 ones to give 22.



Circling the tens and ones that remain will help children to identify how many remain.

They use numberlines and practical resources to support calculation. Teachers *demonstrate* the use of the numberline.

$$6 - 3 = 3$$

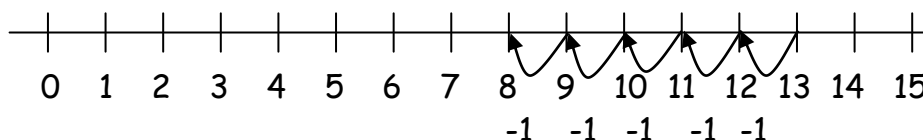


The numberline should also be used to show that $6 - 3$ means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart.



Children then begin to use numbered lines to support their own calculations - using a numbered line to count back in ones.

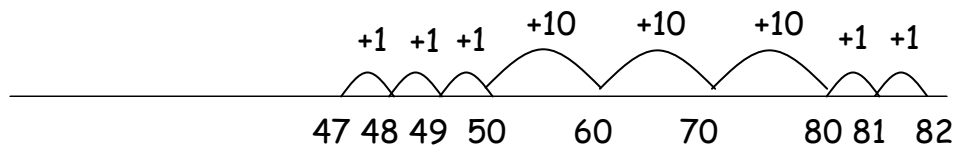
$$13 - 5 = 8$$



Counting on

If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on. Counting on can also be used to solve empty number box calculations.

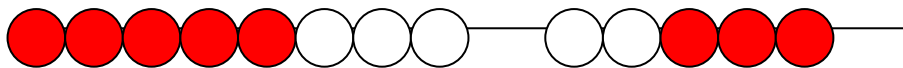
$$82 - 47$$



$$47 + ? = 82$$

Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.

$$13 - 5 = 8$$



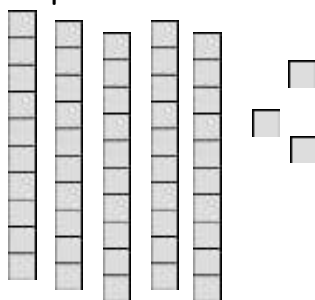
Y2

End of Year Objective:

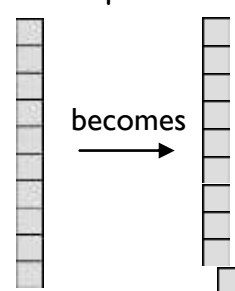
Subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers.

Children continue to use Base 10 equipment to develop their understanding of subtraction. When the amount of ones to be subtracted is greater than the ones in the original number, an exchange method is required. This relies on children's understanding of ten units being an equivalent amount to one ten. To calculate $53 - 26$, by using practical equipment, they would count out 53 using the tens and ones, as in Step 1. They need to consider whether there are enough ones to remove 6. In this case there are not so they need to exchange a ten into ten ones to make sure that there are enough, as in step 2.

Step 1

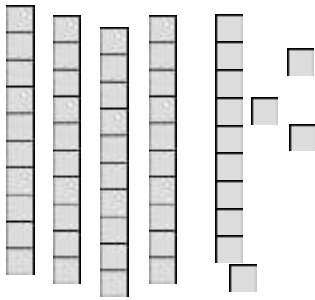


Step 2

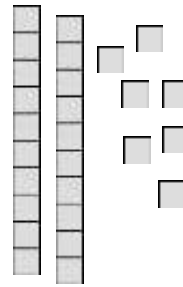


The children can now see the 53 represented as 40 and 13, still the same total, but partitioned in a different way, as in step 3 and can go on to take away the 26 from the calculation to leave 27 remaining, as in Step 4.

Step 3

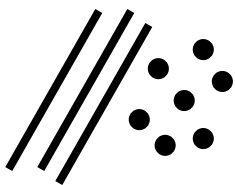


Step 4

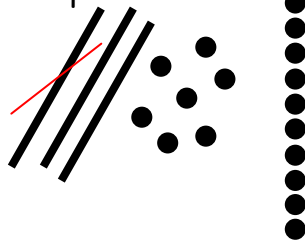


When recording their own drawings, when calculating $37 - 19$, children would cross out a ten and exchange for ten ones. Drawing them in a vertical line, as in Step 2, ensures that children create ten ones and do not get them confused with the ones that were already in place.

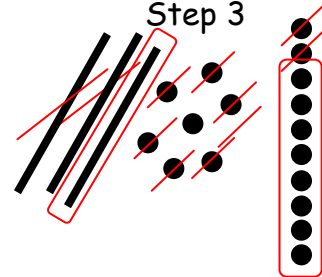
Step 1



Step 2



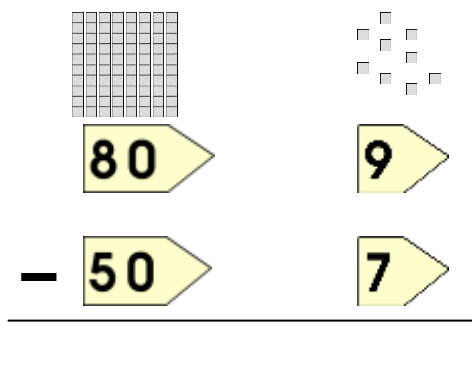
Step 3



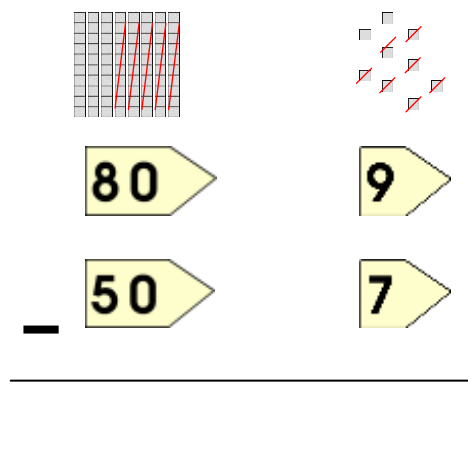
Circling the tens and ones that remain will help children to identify how many remain.

Children will build on their knowledge of using Base 10 equipment from Y2 and continue to use the idea of exchange. This process should be demonstrated using arrow cards to show the partitioning and Base 10 materials to represent the first number, removing the ones and tens as appropriate (as with the more informal method in Y2).

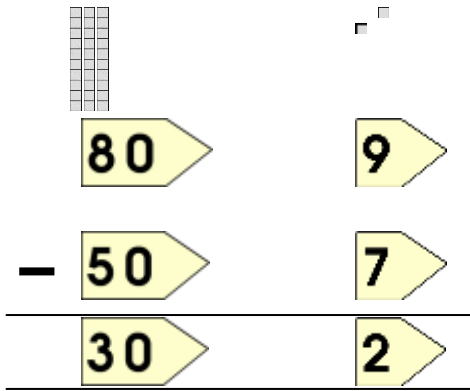
Step 1



Step 2



Step 3

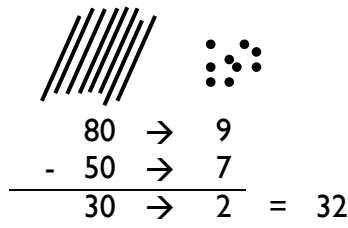


Emphasise that the second (bottom) number is being subtracted from the first (top) number rather than the lesser number from the greater.

This will be recorded by the children as:

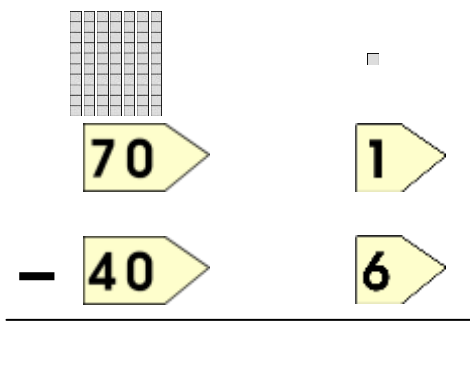
$$\begin{array}{r}
 80 \rightarrow 9 \\
 - 50 \rightarrow 7 \\
 \hline
 30 \rightarrow 2 = 32
 \end{array}$$

Children can also use jottings of the Base 10 materials (as in Year 2) to support with their calculation, as in the example below.

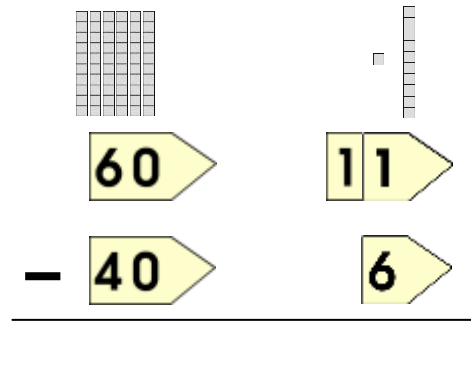


From this the children will begin to solve problems which involve exchange. Children need to consider whether there are enough ones to remove 6. In this case there are not (Step 1) so they need to exchange a ten into ten ones to make sure that there are enough, as they have been doing in the method for Year 2 (Step 2). They should be able to see that the number is just partitioned in a different way, but the amount remains the same ($71 = 70 + 1 = 60 + 11$).

Step 1

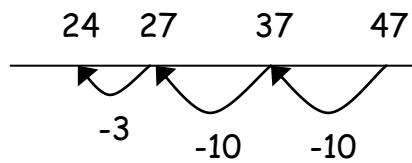


Step 2



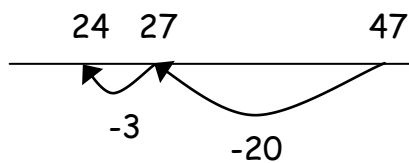
- ✓ Then helping children to become more efficient by subtracting the ones in one jump (by using the known fact $7 - 3 = 4$).

$$47 - 23 = 24$$



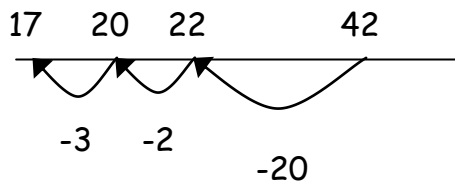
- ✓ Subtracting the tens in one jump and the ones in one jump.

$$47 - 23 = 24$$



- ✓ Bridging through ten can help children become more efficient.

$$42 - 25 = 17$$

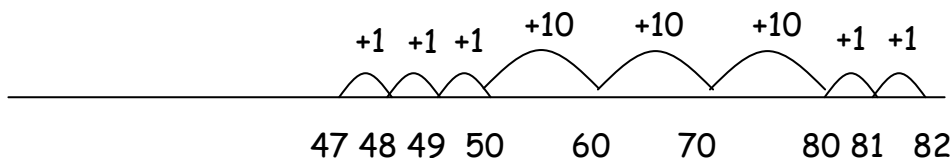


Counting on

If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on. Counting on can also be used to solve empty number box calculations.

Count up from 47 to 82 in jumps of 10 and jumps of 1.

$$82 - 47$$



$$47 + ? = 82$$

Help children to become more efficient with counting on by:

- ✓ Subtracting the ones in one jump;
- ✓ Subtracting the tens in one jump and the ones in one jump;
- ✓ Bridging through ten.

Y3

End of Year Objective:

Subtract numbers with up to three digits, using formal written method of columnar subtraction.*

In year 3, children will continue to subtract using the column method. When introducing the hundreds column, children may use arrow cards and Base 10 equipment to consolidate their understanding. This process might look similar to that of year 2, dependent on the need of the children.

By the end of year 3, children will be able to confidently subtract 3 digit numbers from 3 digit numbers and will be starting to subtract different numbered digits from 3 digits and understanding the place value involved in this.

e.g. $532 - 28$ involves lining up the digits in the correct columns.

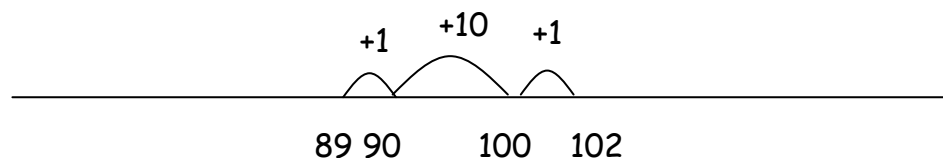
$$\begin{array}{r} 2 \\ 53\cancel{2} \\ - \quad 28 \\ \hline 514 \end{array}$$

Subtraction as finding the difference

If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc., children should be encouraged to recognise that it is more efficient to find the difference by counting up using a number line to support the mental calculation.

e.g.

$$102 - 89 = 13$$



Help children be more efficient with counting on by:

- Subtracting the ones in one jump;
- Subtracting the tens in one jump and the ones in one jump.

Y4

End of Year Objective:

Subtract numbers with up to 4 digits *and decimals with one decimal place* using the formal written method of columnar subtraction where appropriate.

Children in year four will continue to use the concise method of subtraction up to four digits. They will use this method when an exchange has to take place consecutively.

e.g. 4754 requires a double exchange.

$$\begin{array}{r} 4 \overset{6}{\cancel{7}} \overset{14}{\cancel{5}} 4 \\ - 3 \quad 2 \quad 8 \quad 6 \\ \hline 1 \quad 4 \quad 6 \quad 8 \end{array}$$

By the end of Y4, children should be using the written method confidently and with understanding. They will also be subtracting:

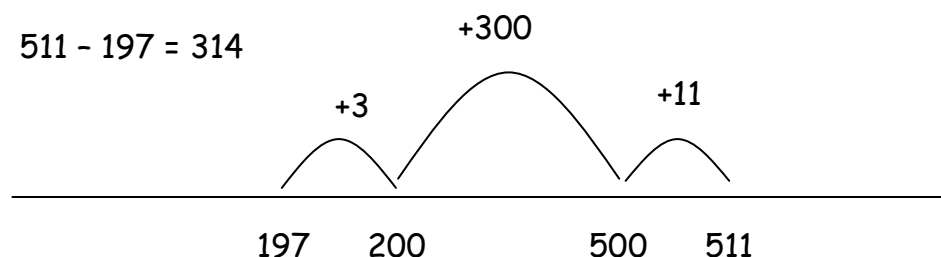
- numbers with different numbers of digits, understanding the place value;
- *decimals with one decimal place, knowing that the decimal points line up under one another.*

Subtraction as finding the difference

If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc., children should be encouraged to recognise that it is more efficient to find the difference by counting up using a number line to support the mental calculation.

e.g.

$$511 - 197 = 314$$



Help children be more efficient with counting on by:

- Subtracting the ones in one jump;
- Subtracting the tens in one jump and the ones in one jump.
- Subtracting the hundreds in one jump, the tens in one jump and the ones in one jump.

Y5

End of Year Objective:

Subtract whole numbers with more than 4 digits *and decimals with two decimal places*, including formal written methods (columnar subtraction).

Children should continue to use the decomposition method to solve calculations such as:

$$\begin{array}{r} \overset{6}{\cancel{7}} \overset{10}{} \overset{6}{\cancel{7}} \overset{12}{} \\ - 3 2 2 6 \\ \hline 3 8 4 6 \end{array}$$

$$\begin{array}{r} \overset{2}{\cancel{3}} . \overset{13}{\cancel{4}} \overset{12}{} \\ - 1 . 7 6 \\ \hline 1 . 6 6 \end{array}$$

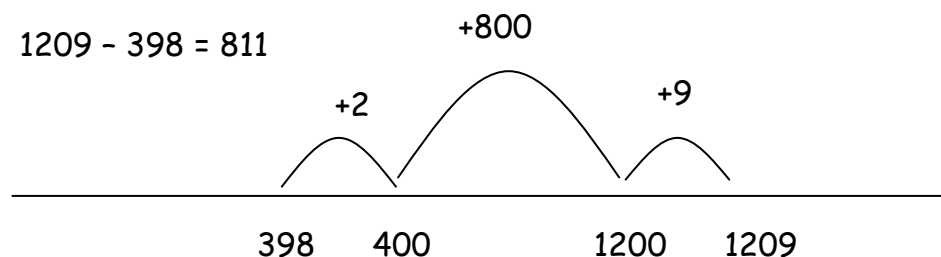
They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with each number having the same number of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another

If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc., children should be encouraged to recognise that it is more efficient to find the difference by counting up using a number line to support the mental calculation.

e.g.

$$1209 - 398 = 811$$



Progression Towards a Written Method for Multiplication

In developing a written method for multiplication, it is important that children understand the concept of multiplication, in that it is:

- repeated addition

They should also be familiar with the fact that it can be represented as an array

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of division
- commutative i.e. 5×3 is the same as 3×5
- associative i.e. $2 \times 3 \times 5$ is the same as $2 \times (3 \times 5)$

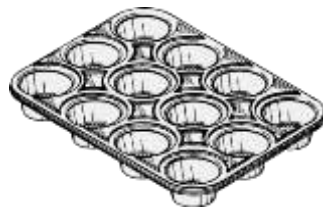
EYFS

Early Learning Goal:

Children solve problems, including doubling.

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

Children may also investigate putting items into resources such as egg boxes, ice cube trays and baking tins which are arrays.

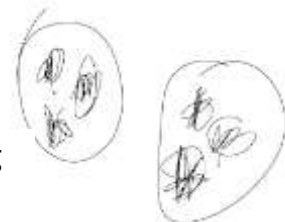


They may develop ways of recording calculations using pictures, etc.



A child's jotting showing the fingers on each hand as a double.

A child's jotting showing double three as three cookies on each plate.



Y1

End of Year Objective:

Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

In year one, children will continue to solve multiplication problems using practical equipment and jottings. They may use the equipment to make groups of objects. Children should see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc and use this in their learning, answering questions such as 'How many eggs would we need to fill the egg box? How do you know?'

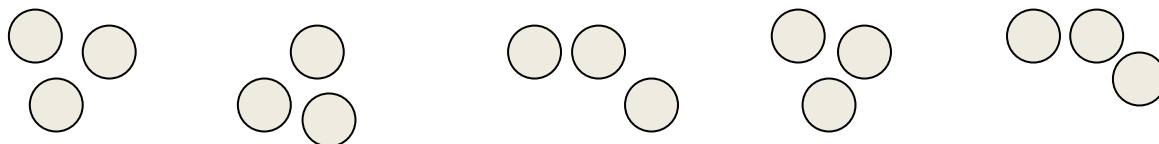
Y2

End of Year Objective:

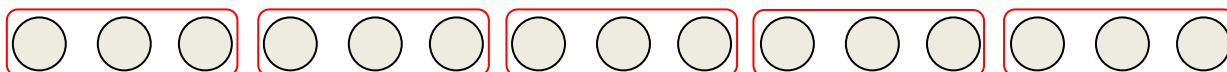
Calculate mathematical statements for multiplication (*using repeated addition*) and write them using the multiplication (x) and equals (=) signs.

Children should understand and be able to calculate multiplication as repeated addition, supported by the use of practical apparatus such as counters or cubes. e.g.

5×3 can be shown as five groups of three with counters, either grouped in a random pattern, as below:

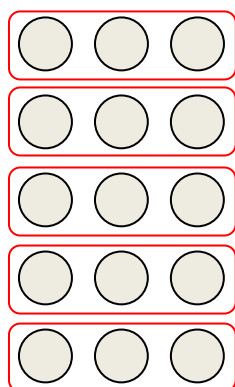


or in a more ordered pattern, with the groups of three indicated by the border outline:

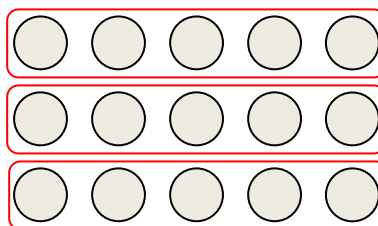


Children should then develop this knowledge to show how multiplication calculations can be represented by an array, (this knowledge will support with the development of the grid method in the future). Again, children should be encouraged to use practical apparatus and jottings to support their understanding, e.g.

$5 \times 3^*$ can be represented as an array in two forms (as it has commutativity):



$$3 + 3 + 3 + 3 + 3 = 15$$



$$5 + 5 + 5 = 15$$

*For mathematical accuracy 5×3 is represented by the second example above, rather than the first as it is five, three times. However, because we use terms such as 'groups of' or 'lots of', children are more familiar with the initial notation. Once children understand the commutative order of multiplication the order is irrelevant).

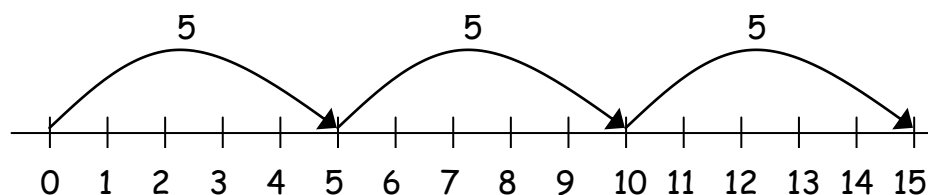
Children will develop their understanding of multiplication and use jottings to support calculation:

✓ **Repeated addition**

5×3 is $5 + 5 + 5 = 15$ or 3 lots of 5

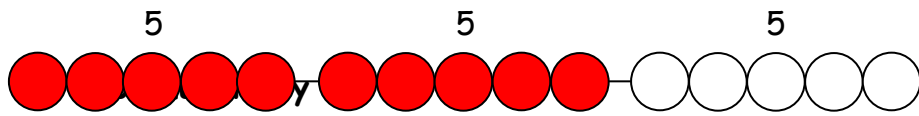
Repeated addition can be shown easily on a number line:

$$5 \times 3 = 5 + 5 + 5$$



and on a bead bar:

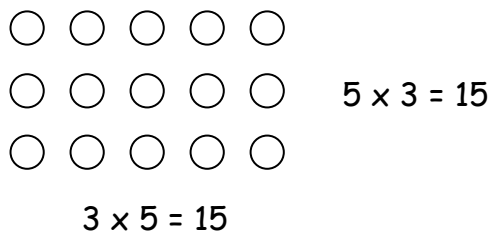
$$5 \times 3 = 5 + 5 + 5$$



Children should know that 3×5 has the same answer as 5×3 .

✓ **Arrays**

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



Y3

End of Year Objective:

Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, progressing to formal written methods.*

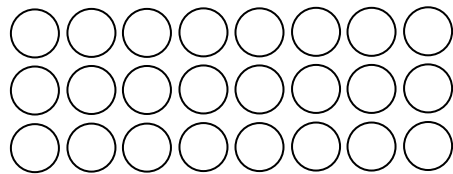
**Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4*

It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

Initially, children will continue to use arrays where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

$$3 \times 8$$

They may show this using practical equipment:



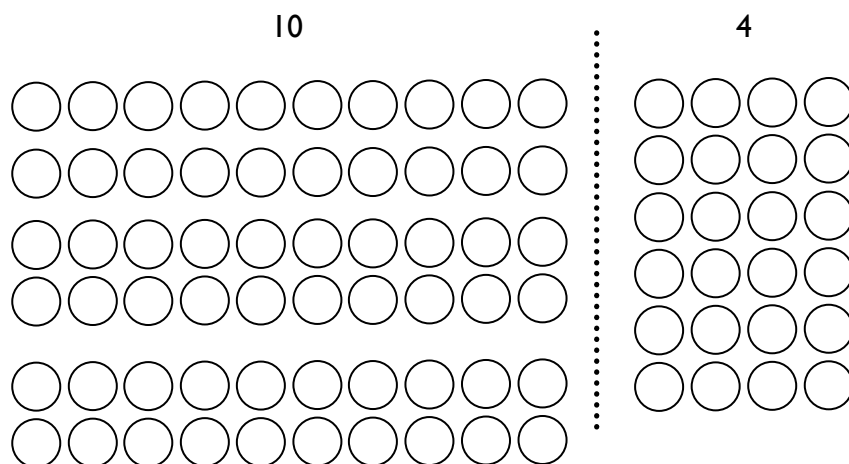
$$3 \times 8 = 8 + 8 + 8 = 24$$

or by jottings using squared paper:

	x	x	x	x	x	x	x	x	
	x	x	x	x	x	x	x	x	
	x	x	x	x	x	x	x	x	

$$3 \times 8 = 8 + 8 + 8 = 24$$

As they progress to multiplying a two-digit number by a single digit number, children should use their knowledge of partitioning two digit numbers into tens and ones/ones to help them. For example, when calculating 14×6 , children should set out the array, then partition the array so that one array has ten columns and the other four.



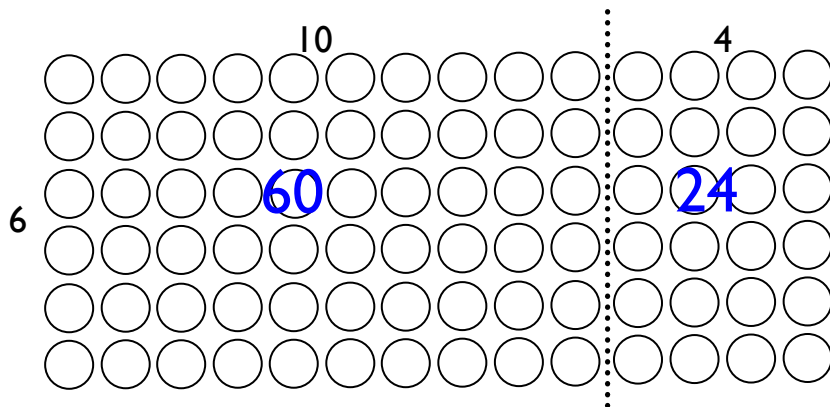
Partitioning in this way, allows children to identify that the first array shows 10×6 and the second array shows 4×6 . These can then be added to calculate the answer:

$$(6 \times 10) + (6 \times 4)$$
$$= 60 + 24$$

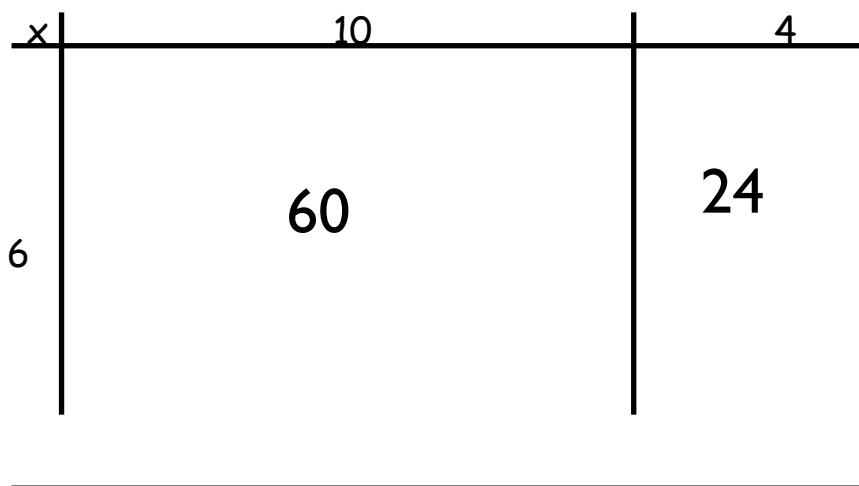
NB There is no requirement for children to record in this way, but it could be used as a jotting to support development if needed.

= 84

This method is the precursor step to the grid method. Using a two-digit by single digit array, they can partition as above, identifying the number of rows and the number of columns each side of the partition line.



By placing a box around the array, as in the example below, and by removing the array, the grid method can be seen.



It is really important that children are confident with representing multiplication statements as arrays and understand the rows and columns structure before they develop the written method of recording.

From this, children can use the grid method to calculate two-digit by one-digit multiplication calculations, initially with two digit numbers less than 20. Children should be encouraged to set out their addition in a column at the side to ensure the place value is maintained. When children are working with numbers where they can confidently and correctly calculate the addition mentally, they may do so.

13×8

x	10	3
8	80	24

$$\begin{array}{r} 80 \\ + 24 \\ \hline 104 \end{array}$$

When children are ready, they can then progress to using this method with other two-digit numbers.

37×6

x	30	7
6	180	42

$$\begin{array}{r} 180 \\ + 42 \\ \hline 222 \end{array}$$

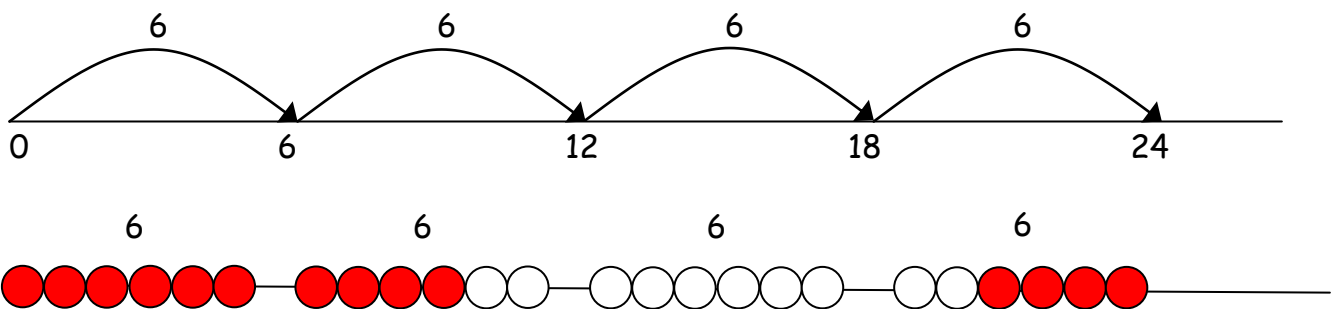
Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

Children will continue to use:

✓ **Repeated addition**

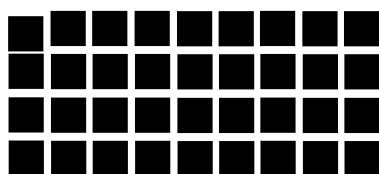
6×4 is $6 + 6 + 6 + 6 = 24$ or 4 lots of 6

Children should use number lines or bead bars to support their understanding.



✓ **Arrays**

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



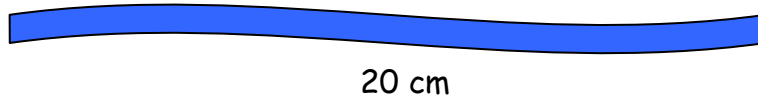
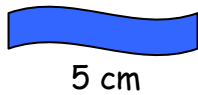
$9 \times 4 = 36$

$9 \times 4 = 36$

Children will also develop an understanding of

✓ **Scaling**

e.g. Find a ribbon that is 4 times as long as the blue ribbon



✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations**

$$\square \times 5 = 20$$

$$3 \times \triangle = 18$$

$$\square \times \bigcirc = 32$$

Y4

End of Year Objective:

Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.

Children will move to Y4 using whichever method they were using as they transitioned from Y3.

They will further develop their knowledge of the grid method to multiply any two-digit by any single-digit number, e.g.

$$79 \times 8$$

	x	70	9
8		560	72

$$\begin{array}{r} 560 \\ + 72 \\ \hline 632 \\ \hline \end{array}$$

To support the grid method, children should develop their understanding of place value and facts that are linked to their knowledge of tables. For example, in the calculation above, children should use their knowledge that $7 \times 8 = 56$ to know that $70 \times 8 = 560$.

By the end of the year, they will extend their use of the grid method to be able to multiply three-digit numbers by a single digit number, e.g.

$$346 \times 8$$

x	300	40	6
8	2400	320	48

$$\begin{array}{r} 2400 \\ + 320 \\ + 48 \\ \hline 2768 \end{array}$$

When children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they may do so.

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

Y5

End of Year Objective:

Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.

Children should continue to use the grid method and extend it to multiplying numbers with up to four digits by a single digit number, e.g.

$$4346 \times 8$$

x	4000	300	40	6
8	32000	2400	320	48

$$\begin{array}{r} 32000 \\ + 2400 \\ + 320 \\ + 48 \\ \hline 34768 \end{array}$$

and numbers with up to four digits by a two-digit number, e.g.

$$2693 \times 24$$

x	2000	600	90	3	
20	40000	12000	1800	60	40000
4	8000	2400	360	12	+ 8000
					+ 12000
					+ 2400
					+ 1800
					+ 360
					+ 60
					+ 12
					<hr/> 64632

When children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they may do so.

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

Y6

End of Year Objective:

Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.

Multiply one-digit numbers with up to two decimal places by whole numbers.

By the end of Y6, children should be able to use the grid method to multiply any number by a two-digit number. They should also develop the method to be able to multiply decimal numbers with up to two decimal places, e.g.

$$4.92 \times 3$$

x	4	0.9	0.02	
3	12	2.7	0.06	12
				+ 2.7
				+ 0.06
				<hr/> 14.76

When children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they may do so.

Children should also be using this method to solve problems and multiply numbers, including those with decimals, in the context of money or measures, e.g. to calculate the cost of 7 items at £8.63 each, or the total length of six pieces of ribbon of 2.28m each.

When ready children may move onto the expanded written method

67.4×8 is equivalent to $674 \times 8 \div 10$

$$\begin{array}{r} 674 \\ \times \quad 8 \\ \hline 32 \quad (4 \times 8) \\ 560 \quad (70 \times 8) \\ \hline 4800 \quad (600 \times 8) \\ \hline 5392 \\ \hline 1 \end{array}$$

Formal written method

The grid method leads children to the compact formal written method so that children understand its structure and efficiency.

The amount of time that should be spent teaching and practising the grid method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

$$\begin{array}{r} 674 \\ \times \underline{53}8 \\ \hline 5392 \end{array}$$

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Expanded written method

The grid method leads children to the expanded written method so that children understand its structure and efficiency.

The amount of time that should be spent teaching and practising the grid method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Multiplying the least significant digit first

$$\begin{array}{r} 284 \\ \times 63 \\ \hline 8^2 5^1 2 \\ 17^5 0^2 40 \\ \hline 17892 \end{array}$$

Formal written method

The expanded written method leads children to the compact formal written method so that children understand its structure and efficiency.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Multiplying the least significant digit first

$$\begin{array}{r} 284 \\ \times 63 \\ \hline 8^2 5^1 2 \\ 17^5 0^2 40 \\ \hline 17892 \end{array}$$

Progression Towards a Written Method for Division

In developing a written method for division, it is important that children understand the concept of division, in that it is:

- repeated subtraction
- sharing into equal amounts

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of multiplication
- not commutative i.e. $15 \div 3$ is not the same as $3 \div 15$
- not associative i.e. $30 \div (5 \div 2)$ is not the same as $(30 \div 5) \div 2$

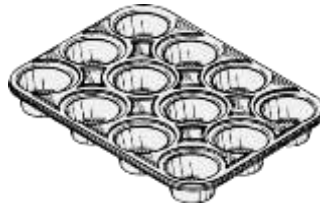
EYFS

Early Learning Goal:

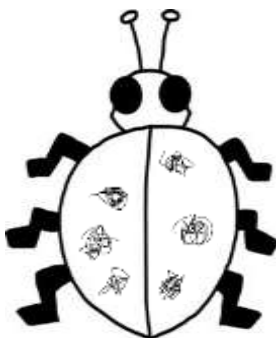
Children solve problems, including halving and sharing.

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

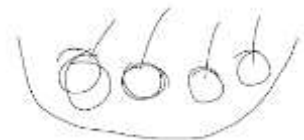
Children may also investigate sharing items or putting items into groups using items such as egg boxes, ice cube trays and baking tins which are arrays.



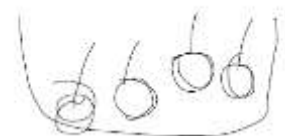
They may develop ways of recording calculations using pictures, etc.



A child's jotting showing halving six spots between two sides of a ladybird.



A child's jotting showing how they shared the apples at snack time between two groups.

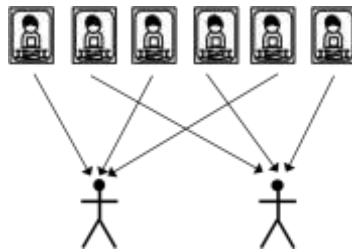


Y1

End of Year Objective:

Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

In year one, children will continue to solve division problems using practical equipment and jottings. They should use the equipment to share objects and separate them into groups, answering questions such as 'If we share these six apples between the three of you, how many will you each have? How do you know?' or 'If six football stickers are shared between two people, how many do they each get?' They may solve both of these types of question by using a 'one for you, one for me' strategy until all of the objects have been given out.



Children should be introduced to the concept of simple remainders in their calculations at this practical stage, being able to identify that the groups are not equal and should refer to the remainder as '... left over'.

Y2

End of Year Objective:

Calculate mathematical statements for division within the multiplication tables and write them using the division (\div) and equals (=) signs.

Children will utilise practical equipment to represent division calculations as grouping (repeated subtraction) and use jottings to support their calculation, e.g.

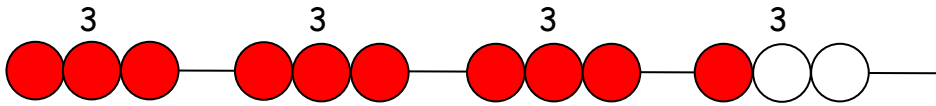
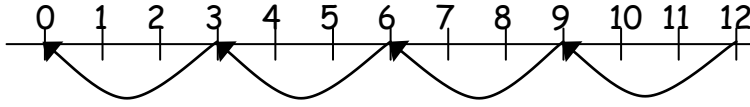
$$12 \div 3 =$$



Children need to understand that this calculation reads as 'How many groups of 3 are there in 12?'

✓ **Repeated subtraction using a number line or bead bar**

$$12 \div 3 = 4$$



The bead bar will help children with interpreting division calculations such as $10 \div 5$ as 'how many 5s make 10?'

✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations**

$$\square \div 2 = 4$$

$$20 \div \triangle = 4$$

$$\square \div \triangle = 4$$

They should also continue to develop their knowledge of division with remainders, e.g.

$$13 \div 4 =$$



$$13 \div 4 = 3 \text{ remainder } 1$$

Children need to be able to make decisions about what to do with remainders after division and round up or down accordingly. In the calculation $13 \div 4$, the answer is 3 remainder 1, but whether the answer should be rounded up to 4 or rounded down to 3 depends on the context, as in the examples below:

I have £13. Books are £4 each. How many can I buy?

Answer: 3 (the remaining £1 is not enough to buy another book)

Apples are packed into boxes of 4. There are 13 apples. How many boxes are needed?

Answer: 4 (the remaining 1 apple still needs to be placed into a box)

Y3

End of Year Objective:

Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, progressing to formal written methods.*

**Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4*

It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

Initially, children will continue to use division by grouping (including those with remainders), where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

$$43 \div 8 =$$

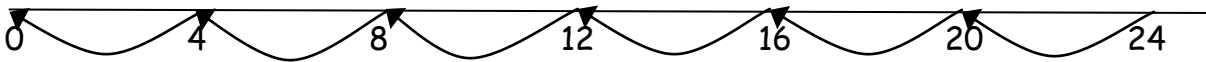


$$43 \div 8 = 5 \text{ remainder } 3$$

Repeated subtraction using a number line

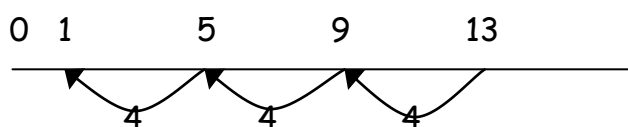
Children will use an empty number line to support their calculation.

$$24 \div 4 = 6$$



Children should also move onto calculations involving remainders.

$$13 \div 4 = 3 \text{ r } 1$$



- ✓ Using symbols to stand for unknown numbers to complete equations using inverse operations

$$26 \div 2 = \square \qquad 24 \div \triangle = 12 \qquad \square \div 10 = 8$$

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

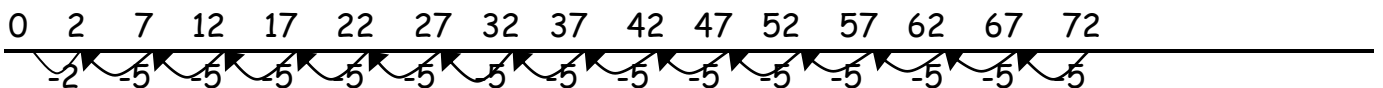
Y4

End of Year Objective:

Divide numbers up to 3 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

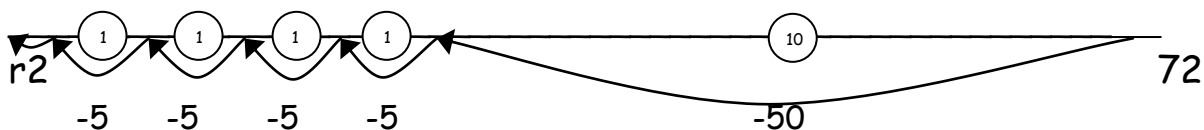
Children will develop their use of repeated subtraction to be able to subtract multiples of the divisor. Initially, these should be multiples of 10s, 5s, 2s and 1s - numbers with which the children are more familiar.

$$72 \div 5$$



$$0 \quad 2 \quad 7 \quad 12 \quad 17 \quad 22$$

Moving onto:



Children will continue to develop their use of grouping (repeated subtraction) to be able to subtract multiples of the divisor, moving on to the use of the 'chunking' method.

$$\begin{array}{r}
 14 \\
 4 \overline{) 56} \\
 \underline{- 40} \\
 16 \\
 \underline{- 16} \\
 0
 \end{array}$$

Children should write their answer above the calculation to make it easy for them and the teacher to distinguish.

Answer: 14

The number line method used in year 3 can be linked to the chunking method to enable children to make links in their understanding.

When developing their understanding of 'chunking', children should utilise a 'key facts' box, as shown below. This enables an efficient recall of tables facts and will help them in identifying the largest group they can subtract in one chunk. Any remainders should be shown as integers, e.g.

$$73 \div 3$$

$$\begin{array}{r}
 24r1 \\
 3 \overline{) 73} \\
 \underline{- 30} \\
 43 \\
 \underline{- 30} \\
 13 \\
 \underline{- 6} \\
 7 \\
 \underline{- 6} \\
 1
 \end{array}$$

Key facts box

1x	3
2x	6
5x	15
10x	30

By the end of year 4, children should be able to use the chunking method to divide a three digit number by a single digit number. To make this method more efficient, the key facts in the menu box should be extended to include 4x and 20x, e.g.

$$196 \div 6$$

$$\begin{array}{r}
 32r4 \\
 6 \overline{) 196} \\
 \underline{- 120} \\
 76 \\
 \underline{- 60} \\
 16 \\
 \underline{- 12} \\
 4
 \end{array}$$

Key facts box

1x	6
2x	12
4x	24
5x	30
10x	60
20x	120

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

Y5

End of Year Objective:

Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

Children may continue to use the key facts box for as long as they find it useful. Using their knowledge of linked tables facts, children should be encouraged to use higher multiples of the divisor. Any remainders should be shown as integers, e.g.

$$523 \div 8$$

$$\begin{array}{r} 65r3 \\ 8 \overline{) 523} \\ - 320 \\ \hline 203 \\ - 160 \\ \hline 43 \\ - 40 \\ \hline 3 \end{array}$$

By the end of year 5, children should be able to use the chunking method to divide a four digit number by a single digit number. If children still need to use the key facts box, it can be extended to include 100x.

$$2458 \div 7$$

$$\begin{array}{r} 351r1 \\ 7 \overline{) 2458} \\ - 2100 \\ \hline 358 \\ - 350 \\ \hline 8 \\ - 7 \\ \hline 1 \end{array}$$

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

Y6

End of Year Objective:

Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.

Use written division methods in cases where the answer has up to two decimal places.

- Divide multi-digit numbers up to 4 digits by a one-digit whole number
- Divide one-digit or two-digit numbers up to two decimal places by a one-digit number

Whole numbers

$$\begin{array}{r} \underline{262R4} \\ 7 \overline{)18^43^18} \end{array}$$

Decimals

1. Calculating with decimals

$$\begin{array}{r} \underline{7.56} \\ 6 \overline{)45.^33^36} \end{array}$$

2. Converting decimals to whole numbers before calculating, then converting the answer back to decimals

45.36 ÷ 6 is equivalent to 4536 ÷ 6 ÷ 100

$$\begin{array}{r} \underline{756} \\ 6 \overline{)45^33^36} \end{array}$$

$$756 \div 100 = 7.56$$

To develop the chunking method further, it should be extended to include dividing a four-digit number by a two-digit number, e.g.

$$6367 \div 28$$

$$\begin{array}{r} \underline{227r11} \\ 28 \overline{)6367} \\ - \underline{5600} \\ \quad 767 \\ - \underline{560} \\ \quad \quad 207 \\ - \underline{140} \\ \quad \quad \quad 67 \\ - \underline{56} \\ \quad \quad \quad \quad 11 \end{array}$$

200x
20x
5x
2x

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

In addition, children should also be able to use the chunking method and solve calculations interpreting the remainder as a decimal up to two decimal places, e.g. $362 \div 17$

$362 \div 17$

$$\begin{array}{r}
 21.29 \\
 17 \overline{)362} \\
 - 340 \\
 \hline
 22 \\
 - 17 \\
 \hline
 5.0 \\
 - 3.4 \\
 \hline
 1.60 \\
 - 1.53 \\
 \hline
 0.07
 \end{array}$$

20x
1x
0.2x
0.09x

To enable children to continue the calculation, they need to understand that 5 is the same as 5.0

When recalling and deriving multiplication and division facts, children should also identify decimal equivalents of times tables, e.g. if $2 \times 17 = 34$, I know that $0.2 \times 17 = 3.4$ if $9 \times 17 = 153$, $0.9 \times 17 = 15.3$ so $0.09 \times 17 = 1.53$

For simple fraction and decimal equivalents, this could also be demonstrated using a simple calculation such as $13 \div 4$ to show the remainder initially as a fraction.



Using practical equipment, children can see that for $13 \div 4$, the answer is 3 remainder 1, or put another way, there are three whole groups and a remainder of 1. This remainder is one part towards a full group of 4, so is $\frac{1}{4}$. To show the remainder as a fraction, it becomes the numerator where the denominator is the divisor (the number that you are dividing by in the calculation).

$$3574 \div 8$$

$$\begin{array}{r} 8 \overline{) 3574} \\ - 3200 \\ \hline 374 \\ - 320 \\ \hline 54 \\ - 48 \\ \hline 6 \end{array}$$

400x
40x
6x

$$\frac{6}{8}$$

← remainder
← divisor

So $3574 \div 8$ is $446\frac{6}{8}$
(when the remainder is shown as a fraction)

To show the remainder as a decimal relies upon children's knowledge of decimal fraction equivalents. For decimals with no more than 2 decimal places, they should be able to identify:

Half: $\frac{1}{2} = 0.5$

Quarters: $\frac{1}{4} = 0.25$, $\frac{3}{4} = 0.75$

Fifths: $\frac{1}{5} = 0.2$, $\frac{2}{5} = 0.4$, $\frac{3}{5} = 0.6$, $\frac{4}{5} = 0.8$

Tenths: $\frac{1}{10} = 0.1$, $\frac{2}{10} = 0.2$, $\frac{3}{10} = 0.3$, $\frac{4}{10} = 0.4$, $\frac{5}{10} = 0.5$, $\frac{6}{10} = 0.6$, $\frac{7}{10} = 0.7$, $\frac{8}{10} = 0.8$, $\frac{9}{10} = 0.9$

and reduce other equivalent fractions to their lowest terms.

In the example above, $3574 \div 8$, children should be able to identify that the remainder as a fraction of $\frac{6}{8}$ can be written as $\frac{3}{4}$ in its lowest terms. As $\frac{3}{4}$ is equivalent to 0.75, the answer can therefore be written as 446.75